répartitions de températures dans la sous-couche visqueuse; les relations présentées par ces auteurs sont les suivantes:

Reichardt: $\varepsilon_M = \varepsilon_H$; $\dfrac{\varepsilon_M}{\nu} = 3.10^{-5}\,y^{+5}$ pour $y^+ < 5$

Bocquillon: $\varepsilon_M = \varepsilon_H$; $\dfrac{\varepsilon_M}{\nu} = 1{,}6.10^{-3}\,y^{+3}$ pour $y^+ < 10$.

Les courbes ainsi obtenues sont tracées sur la Fig. 4 avec nos résultats expérimentaux et l'on voit très bien que ces expressions théoriques ne permettent pas de représenter la réalité physique avec une précision suffisante. Dans le cas de Reichardt les calculs ultérieurs ont ainsi nécessairement sous-estimé les coefficients de transfert aux grands nombres de Prandtl. Et c'est justement pour $Pr \gg 1$ que l'influence de ce nombre devient prépondérante étant donné que la couche-limite thermique est essentiellement immergée dans la sous-couche visqueuse. Ainsi une erreur sur la valeur de la diffusivité turbulente gagne de l'importance quand le nombre de Prandtl est grand. Cette erreur n'aura pas les mêmes conséquences quant aux calculs des coefficients de transfert thermique, s'il s'agit des métaux liquides, car la conductibilité thermique est beaucoup plus élevée et une erreur sur la valeur de la diffusivité thermique turbulente ne se fera que très peu remarquer.

RÉFÉRENCES

1. J. C. NEUMANN, Etude du transfert thermique aux grands nombres de Prandtl, Thèse, Fac. des Sciences de Paris (Nov. 1967).
2. H. REICHARDT, Die Wärmeübertragung in turbulenten Reibungsschichten. *Z.A.M.M.* **20**, Heft 6, 297 (1940).
3. C. BOCQUILLON: Sur les propriétés mécaniques et thermiques des écoulements turbulents au voisinage des parois chauffées. La Houille Blanche, No. 3, 389–395 (1964).

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ENERGY TRANSFER MECHANISMS IN TRANSITION POOL BOILING

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NOMENCLATURE

f_c, liquid–solid contact frequency;
k, thermal conductivity;
q_c, heat flux based on quenching model;
q_B, total boiling heat flux;
Q_c, heat flux to the liquid per contact;
t_c, contact time during which surface temperature drops;
ΔT_c, surface temperature drop by quenching;
ΔT_s, surface temperature difference; surface temperature minus saturated temperature of liquid;
α, thermal diffusivity.

Subscripts
l, liquid;
s, solid.

INTRODUCTION

IN THE regime of transition boiling the anomalous condition exists wherein an increase in temperature difference between a hot surface and an adjacent saturated liquid results in a decrease in heat transfer. Experimental work of Berenson [1], confirmed by others [2–4] supports the existence of solid–liquid contact during transition boiling. Berenson concluded that unstable film boiling and unstable nucleate boiling occur alternately at a given location on the boiling surface and that the variation in heat flux with temperature is associated with different fractions of time each boiling regime exists. Bankoff and Mehra [5] proposed a quenching theory for transition boiling assuming that substantially all of the heat transfer occurs by periodic quenching of the solid surface when directly contacted by the liquid. During the short time of liquid–solid contact, assuming a linear flow of heat in an infinite composite solid, the expression for heat flux to the liquid per contact is

$$Q_c = 2k_l \Delta T_{cl} \left(\frac{t_c}{\pi \alpha_l}\right)^{\frac{1}{2}} \tag{1}$$

where

$$\Delta T_{cl} = \left[1 + \frac{k_l}{k_s}\left(\frac{\alpha_s}{\alpha_l}\right)\right]^{-1} \Delta T_c. \tag{2}$$

Introduction of the mean contact frequency, f_c, yields for the total heat flux

$$q_B = f_c Q_c. \tag{3}$$

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Bankoff and Mehra used values of f_c obtained by Zuber and Tribus [6] who considered wave motion at a liquid vapor interface.

This paper describes work in which the transition boiling regime was studied experimentally at atmospheric pressure and the findings explained in terms of a revised quenching model.

EXPERIMENTAL APPARATUS

Stable transition boiling may be attained in the laboratory if the temperature difference between the boiling surface and the adjacent saturated liquid is controlled rather than the heat flux. The experimental apparatus used in this study is shown schematically in Fig. 1. The water vapor, used to supply heat to the boiling surface, condensed on the thin copper fins (a) connected to the bottom of the boiling surface (b) which was also copper 2 in. dia. and $\frac{3}{32}$ in. thick. Saturation pressure in the pressure vessel (c) was controlled to achieve a desired boiling surface temperature. In the boiler (d) n-pentane, having a vapor pressure of 1 atmosphere at 97°F, was used for transition boiling studies. Heat flux values were determined from the rate of n-pentane condensate accumulation corrected with appropriate heat loss allowances obtained from separate experiments. Two thermocouples were imbedded in the boiling surface; one was used to determine the average surface temperature and the other to measure surface temperature fluctuations at a point. The thermocouple measuring surface temperature fluctuation was connected to an oscilloscope with a differential preamplifier. [7] may be consulted for a more complete description of the apparatus.

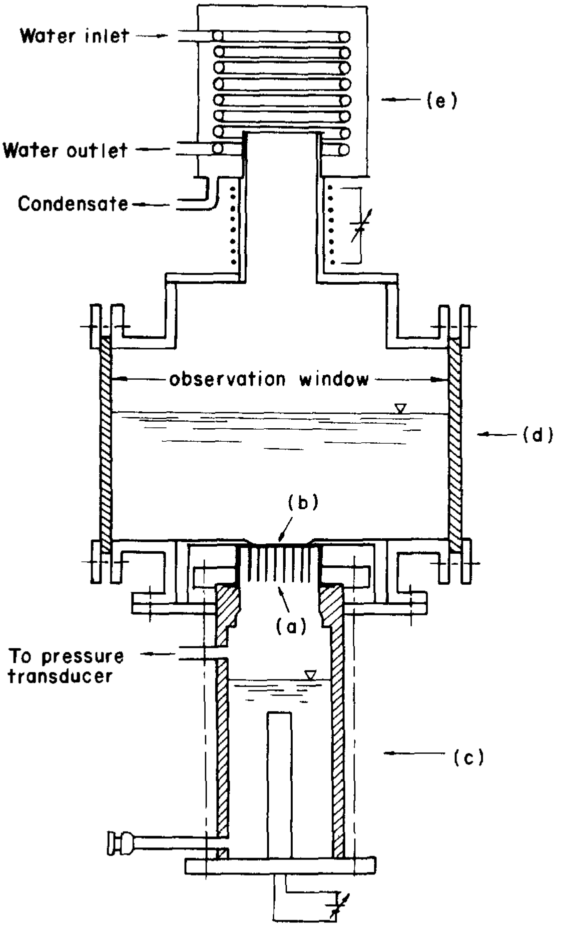

FIG. 1. Schematic of experimental apparatus.

RESULTS AND DISCUSSION

When saturated liquid strikes a hot surface with a given velocity a pressure above the saturation value is reached and maintained for a finite length of time, t_c. During this short time interval liquid in contact with the surface is subcooled and energy is transferred rapidly to the liquid through a very thin liquid film at the liquid–solid interface resulting in a rapid decrease in surface temperature of magnitude ΔT_c. This temperature decrease was obtained from the oscilloscope trace. A linear regression analysis of 40 data points yielded the expression

$$\Delta T_c = 0{\cdot}379\,\Delta T_s + 0{\cdot}363 \quad [°\mathrm{F}] \tag{4}$$

and with a 50 per cent confidence level the upper and lower limits respectively were

$$\Delta T_c = 0{\cdot}418\,\Delta T_s + 3{\cdot}83 \quad [°\mathrm{F}] \tag{5}$$

$$\Delta T_c = 0{\cdot}340\,\Delta T_s - 3{\cdot}16 \quad [°\mathrm{F}]. \tag{6}$$

From these 40 observations, the time t_c during which the surface temperature decreased was observed to be independent of ΔT_s and was approximately equal to 2 ms. A typical oscilloscope trace of surface temperature variation with time is shown in Fig. 2a.

Values of liquid–solid contact frequency, f_c, were obtained from temperature fluctuation data obtained using both an electronic recorder and an oscilloscope. In Fig. 3 contact frequency is plotted vs. the logarithmic value of boiling heat flux, log q_B. The contact frequency approached zero in the neighborhood of the minimum heat flux; for the quenching theory to be valid this is a necessary condition.

Having measured values of ΔT_c, t_c and f_c, the transition boiling heat flux was calculated using a modified form of the Bankoff and Mehra quenching model. The temperature difference being measured was that of the solid surface, ΔT_c, rather than of the liquid, thus k_l and α_l in equations (1) and (2) were replaced by k_s and α_s, respectively. Hence

$$q_c = f_c Q_c = 2 f_c k_s \Delta T_c \left(\frac{t_c}{\pi \alpha_s}\right)^{\frac{1}{2}}. \tag{7}$$

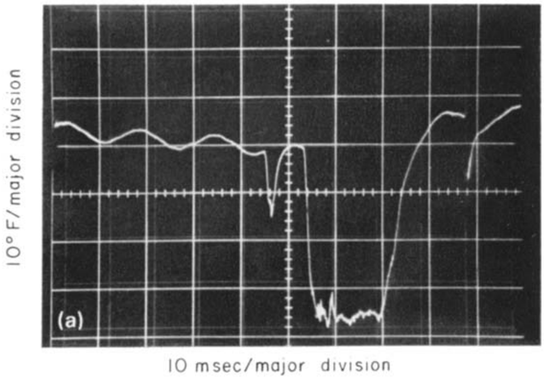

FIG. 2a. Typical oscillograph showing surface temperature fluctuation during transition boiling. $\Delta T_c = 31$°F, $q_B = 1{\cdot}9 \times 10^4$ Btu/h ft^2.

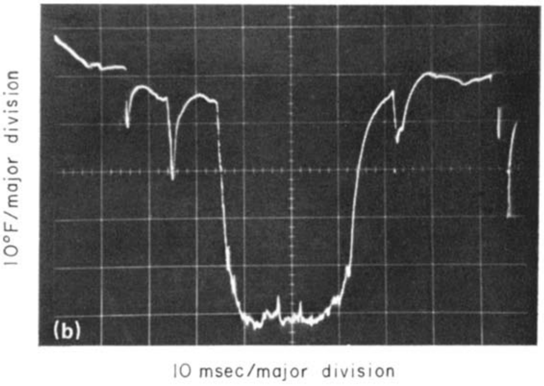

FIG. 2b. Typical oscillograph showing surface temperature fluctuations with n-pentane in the spheroidal state. $\Delta T_s = 92$°F.

In Fig. 4 heat flux, q_c, calculated from equation (7) and total boiling heat flux evaluated from the rate of condensate accumulation are plotted for purposes of comparison. In order to present the heat flux, q_c, over the entire range of transition boiling, it was necessary to extrapolate data obtained at lower heat fluxes. Equation (4) was used in calculating values of ΔT_c at the higher heat fluxes. The vertical line drawn at each value of q_B includes values within the 50 per cent confidence limits. The contribution of quenching to the total heat flux during transition boiling decreases with increasing heat flux; near the burnout point approximately 50 per cent of the heat removal is due to quenching. Therefore, although quenching is a major mechanism for heat exchange in transition boiling, a mechanism other than quenching exists at higher heat flux levels.

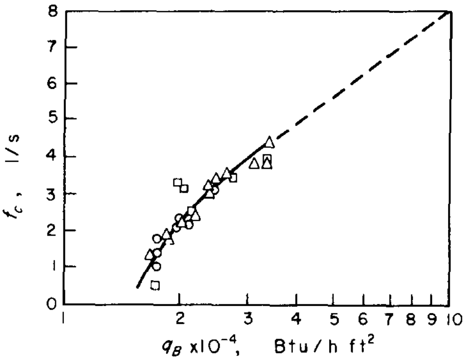

FIG. 3. Correlation of contact frequency, f_c and total boiling heat flux, q_B.

ΔT_s (°F)	$q_B \times 10^{-4}$ (Btu/h ft²)	f_c (1/s)	ΔT_s (°F)	$q_B \times 10^{-4}$ (Btu/h ft²)	f_c (1/s)
	(Exp. #TF 1) ○			(Exp. #TF 8) △	
115·9	1·74	1·0	99·5	1·68	1·27
111·2	1·76	1·4	97·8	1·88	1·73
108·5	1·74	1·8	97·7	1·85	1·80
104·0	1·86	1·8	95·5	1·99	2·20
99·4	1·97	2·2	93·8	2·16	2·30
95·0	2·01	2·4	91·4	2·38	3·00
92·3	2·13	2·2	89·9	2·36	3·15
84·8	2·45	3·2	89·0	2·49	3·40
49·7	8·80	—	86·9	2·65	3·50
			83·2	3·10	3·80
	(Exp. #TF 7) □		79·9	3·43	4·40
94·7	1·74	0·50	81·5	3·40	3·80
89·6	2·09	3·14	50·6	7·92	—
90·7	1·97	3·33	51·8	8·29	—
85·9	2·15	2·43	51·9	8·45	—
77·2	2·77	3·43			
68·5	3·42	4·00			

Oscilloscope traces of surface temperature fluctuations indicated that a sharp drop in temperature was always followed by small but rapid temperature fluctuations. Separate experiments were undertaken wherein liquid pentane was dropped on the bare boiling surface and oscilloscope traces again observed. In these experiments the liquid pentane was in the spheroidal state; i.e. liquid contacted the hot surface without wetting it and also without losing a significant amount of mass by evaporation. A typical oscilloscope trace for these experiments is shown in Fig. 2b.

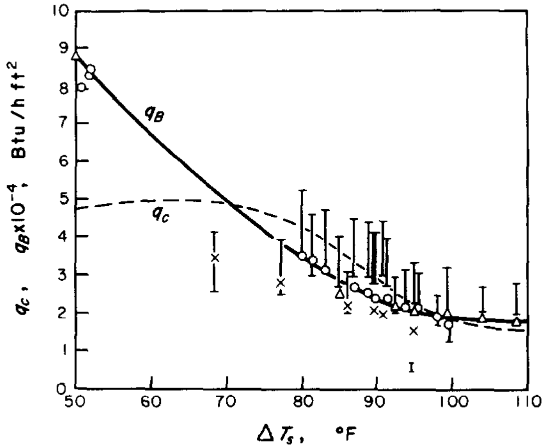

FIG. 4. Contribution of quenching to transition pool boiling.

A striking similarity is observed between temperature fluctuations for these experiments and for the portion of transition boiling following the rapid temperature drop. It is concluded that the liquid exists in the spheroidal state during part of the time transition boiling occurs.

A sharp rise in surface temperature occurred following the spheroidal state of the liquid; a few noticeable peaks were observed during the time the spheroidal state existed. This is interpreted as a momentary covering of a point on the surface by vapor with its insulating effect apparent in the rapid temperature rise at that point. As the surface temperature decreased (accompanied by an increase in heat flux) the frequency of this effect increased. It may be observed that, with the liquid in the spheroidal state, the boiling surface temperature was approximately 30°F below the mean value. In this state, at the higher heat fluxes, conditions are then favorable for the existence of nucleate boiling. In n-pentane the maximum heat flux condition (burnout) occurs at a surface temperature approximately 50°F above that of the saturated liquid. Thus, at surface temperatures from 50 to 80°F above saturation, nucleate boiling will likely occur. The presence of nucleate boiling for values of ΔT_s less than 80°F is, therefore, responsible for the rapid increase in heat flux as the surface temperature is decreased. The decreasing proportion of heat transfer by the quenching mechanism may also be explained by the increased presence of nucleate boiling.

REFERENCES

1. P. J. BERENSON, Experiments on pool boiling heat transfer, *Int. J. Heat Mass Transfer* **5**, 985 (1962).
2. S. ISHIGAI and T. KUNO, An experimental study of transition boiling of water on the vertical wall in an open vessel, *Trans. Japan Soc. Mech. Engrs* **31**, 1251 (1965).
3. K. NISHIKAWA, S. HASEGAWA, H. HONDA and S. SAKAGUCHI, Studies of boiling characteristic curve (2nd rept. of relation between the surface temperature fluctuation and heat transfer), *Trans. Japan Soc. Mech. Engrs* **34**, 152 (1967).
4. R. C. KESSERLING, P. H. ROSCHE and S. G. BANKOFF, Transition and film boiling from horizontal strips. *A.I.Ch.E. Jl* **13**, 669 (1967).
5. S. G. BANKOFF and V. S. MEHRA, A quenching theory for transition boiling, *I/E/C Fundamentals* **1**, 38 (1962).
6. N. ZUBER and M. TRIBUS, Department of Engineering, University of California at Los Angeles, Report No. 58-5 (1958).
7. T. AOKI, An experimental investigation of transition pool boiling, PhD thesis, Oregon State University, Dept. of Mechnical Engineering, Corvallis, Oregon (1970).

Int. J. Heat Mass Transfer. Vol. 13, pp. 1240–1243. Pergamon Press 1970. Printed in Great Britain

ON THE NONLINEARITY OF TWO-DIMENSIONAL HEAT TRANSFER IN A CONDUCTING AND RADIATING MEDIUM*

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NOMENCLATURE

h_s,	$3\epsilon_s/2(2 - \epsilon_s)$;
k,	thermal conductivity;
l,	$\varkappa L$, optical thickness of medium;
n,	inward normal at boundary;
N,	$k\varkappa/(4\sigma T_0^{*3})$;
p,	arccos (T_l/T_0);
$\vec{q}^*$,	heat flux;
$\vec{q}$,	$\vec{q}^*/\sigma T_0^{*4}$;
T^*,	temperature;
T,	T^*/T_0^*;
x^*, y^*,	physical coordinates;
x,	$\varkappa x^*$, $y = \varkappa y^*$, optical coordinates;
∇^2,	$\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$;
ϵ,	emissivity;
$\varkappa$,	absorption coefficient;
σ,	Boltzmann constant;
χ^*,	$k\varkappa\nabla^2 T^* - 3k\varkappa T^* - 4\sigma T^{*4}$;
χ,	$\chi^*/(4\sigma T_0^{*4})$;
ϕ,	$-(\chi + 3NT)$, radiation potential.

Subscripts

s,	value on boundary;
0,	value at x, $y = 0$;
l,	value at x, $y = l$.

INTRODUCTION

THE PROBLEM of energy transfer in a conducting, absorbing and emitting medium has been considered highly non-linear [1], due to the presence of the re-emission term (T^4 for a gray medium). However, for a one dimensional problem it has been shown by Chang [2] that this non-linearity is not as severe as was thought and the re-emission term T^4 can be linearized by functions which can be chosen in a number of ways. In this note a two-dimensional problem, illustrated in Fig. 1, is analyzed according to the differential formulation. The basic non-linear equation is first solved numerically and this solution is then compared to the solution of the linearized equation. The following simplifying assumptions are used: (i) local thermodynamic equilibrium

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